Complex Interpolation of Besov, Modulation and E^λ-spaces

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Besov spaces $B^s_{p,q}(\mathbb{R}^n)$, where $s \in \mathbb{R}$ and $1 \leq p, q \leq \infty$, were introduced in 1959 by O.V. Besov. They consist of tempered distributions whose Besov norm is finite. To compute $\|f\|_{B^s_{p,q}}$ one smoothly decomposes f into parts $\Delta_k f$ whose Fourier support lies in dyadic rings of radii $\sim 2^k$. Then, $\|f\|_{B^s_{p,q}} := \left\| \left(2^{ks} \|\Delta_k f\|_{L_p} \right)_{k \in \mathbb{N}_0} \right\|_{\ell_q}$.

Modulation spaces $M^s_{p,q}(\mathbb{R}^n)$ are constructed similarly. They were introduced in 1983 by H.G. Feichtinger. In contrast to Besov spaces the Fourier support of the parts $\Box_k f$ lie in balls of a uniform radius and centered around $k \in \mathbb{Z}^n$. The norm is given by $\|f\|_{M^s_{p,q}} := \left\| \left((1 + |k|^2)^{s/2} \|\Box_k f\|_{L_p} \right)_{k \in \mathbb{Z}^n} \right\|_{\ell_q}$, i.e., the weight 2^k is replaced by $(1 + |k|^2)^{1/2}$.

E^λ-spaces $E^\lambda_{p,q}(\mathbb{R}^n)$ were introduced by B. Wang in 2006 and use the decomposition of Modulation spaces and the weight of Besov spaces.

In this talk, we introduce a framework describing all of the spaces above and their complex interpolation. To the best of the author's knowledge, the interpolation for E^λ-spaces is not to be found in the literature.