Sampling recovery of functions via ℓ_1-minimization

Tino Ullrich

Using techniques developed recently in the field of compressed sensing we prove new upper bounds for general (non-linear) sampling numbers of (quasi-)Banach smoothness spaces in L_2. In relevant cases such as mixed and isotropic weighted Wiener classes or Sobolev spaces with mixed smoothness, sampling numbers in L_2 can be upper bounded by best n-term trigonometric widths in L_∞. We describe a recovery procedure based on ℓ_1-minimization (basis pursuit denoising) using only m function values. With this method, a significant gain in the rate of convergence compared to recently developed linear recovery methods is achieved. In this deterministic worst-case setting we see an additional speed-up of $m^{-1/2}$ compared to linear methods in case of weighted Wiener spaces. For their quasi-Banach counterparts even arbitrary polynomial speed-up is possible. Surprisingly, our approach allows to recover mixed smoothness Sobolev functions belonging to S^r_pW on the d-torus with a logarithmically better rate of convergence than any linear method can achieve when $1 < p < 2$ and d is large. This effect is not present for isotropic Sobolev spaces.