Runge approximation results and linear topological invariants for kernels of partial differential operators

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Runge's classical approximation theorem characterizes the pairs of open subsets $X_1 \subseteq X_2$ of the complex plane, for which every holomorphic function on X_1 can be approximated by holomorphic functions on X_2 with respect to the compact-open topology. The celebrated Lax-Malgrange theorem from the 1950s, generalizes this approximation result for holomorphic functions, i.e. functions in the kernel of the Cauchy-Riemann operator, to kernels of arbitrary elliptic constant coefficient differential operators. In the past few years, there has been a considerable renewed interest in Runge approximation results, in particular, in quantitative variants.

We discuss quantitative Runge type approximation results for spaces of smooth zero solutions for several classes of constant coefficients linear partial differential operators. Among others, results for arbitrary operators on convex sets, elliptic operators, parabolic operators, and the wave operator in one spatial variable are given. The presented method is based on qualitative approximation results on the one hand and recent results on linear topological invariant of (Ω)-type for kernels of partial differential operators on the other hand. The talk is based on joint work with Andreas Debrouwere (Vrije Universiteit Brussel).

References

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